MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION
(Autonomous)
(ISO/IEC - 27001 - 2013 Certified)

SUMMER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code:** 22224

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	If $f\left(x\right)=x^3-5x^2-4x+20$ show that $f\left(0\right)=-2f\left(3\right)$	**02**
	Ans	$f\left(x\right)=x^3-5x^2-4x+20$	
		$\therefore f\left(0\right)=\left(0\right)^3-5\left(0\right)^2-4\left(0\right)+20=20$	½
		$\therefore f\left(3\right)=\left(3\right)^3-5\left(3\right)^2-4\left(3\right)+20$	½
		$=-10$	
		$\therefore -2f\left(3\right)=-2\times-10=20=f\left(0\right)$	1
		--	
	b)	State whether the function $f(x)=x^3-3x+\sin x+x\cos x,$ is odd or even.	**02**
	Ans	$f(x)=x^3-3x+\sin x+x\cos x$	
		$\therefore f(-x)=\left(-x\right)^3-3\left(-x\right)+\sin\left(-x\right)+\left(-x\right)\cos\left(-x\right)$	½
		$=-x^3+3x-\sin x-x\cos x$	½
		$=-\left(x^3-3x+\sin x+x\cos x\right)$	½
		$=-f\left(x\right)$	
		$\therefore$ Given function is odd.	½
		--	
	c)	If $y=\sin x.\cos 2x,$ find $\frac{dy}{dx}$	**02**
	Ans	$y=\sin x.\cos 2x$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	$\therefore \frac{dy}{dx}=\sin x\left(-\sin 2x\right)\times 2+\cos 2x\cos x$	02
		$=-2\sin x\sin 2x+\cos 2x\cos x$	
	d)	Evaluate: $\int \cos^2 x dx$	**02**
	Ans	$\int \cos^2 x dx=\int \frac{1+\cos 2x}{2}dx$	1
		$=\frac{1}{2}\int\left(1+\cos 2x\right)dx$	
		$=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$	1
	e)	Evaluate: $\int \frac{1}{3x+5}dx$	**02**
	Ans	$\int \frac{1}{3x+5}dx$	
		$=\frac{1}{3}\log\left(3x+5\right)+c$	02
	f)	Find the area between the the line $y=2x$,x-$axis$ and ordinates $x=1$ to $x=3$.	**02**
	Ans	Area $A=\int_a^b y\ dx$	
		$=\int_1^3 2xdx$	½
		$=2\left[\frac{x^2}{2}\right]_1^3$ or $\left[x^2\right]_1^3$	½
		$=2\left[\frac{9}{2}-\frac{1}{2}\right]$ or $\left[3^2-1^2\right]$	½
		$=8$	½
	g)	Find approximate root of the equation $x^2+x-3=0$ in $\left(1,2\right)$ by using Bisection method. $($Use two iterations$)$	**02**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	g)Ans	Let $f\left(x\right)=x^2+x-3$	
		$f\left(1\right)=-1$	
		$f\left(2\right)=3$	
		$\therefore$ the root is in $\left(1,2\right)$	½
		$x_1=\dfrac{a+b}{2}=\dfrac{1+2}{2}=1.5$	½
		$f\left(1.5\right)=0.75>0$	½
		$x_2=\dfrac{x_1+a}{2}=\dfrac{1.5+1}{2}=1.25$	½
		OR	
		Let $f\left(x\right)=x^2+x-3$	
		$f\left(1\right)=-1,\ f\left(2\right)=3$ $\quad\therefore$ the root is in $\left(1,2\right)$	½

Iteration	a	b	$x=\dfrac{a+b}{2}$	$f\left(x\right)$
I	1	2	1.5	0.75
II	1	1.5	1.25	

1½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.		**Solve any <u>THREE</u> of the following :**	**12**
	a)	Find $\dfrac{dy}{dx}$ if $x^3+xy^2=y^3+yx^2$	**04**
	Ans	$x^3+xy^2=y^3+yx^2$	
		$x\left(x^2+y^2\right)=y\left(y^2+x^2\right)$	1
		$x=y$	1
		$\dfrac{dy}{dx}=1$	2
		OR	
		$x^3+xy^2=y^3+yx^2$	
		$3x^2+2xy\dfrac{dy}{dx}+y^2=3y^2\dfrac{dy}{dx}+2xy+x^2\dfrac{dy}{dx}$	2
		$\dfrac{dy}{dx}\left(2xy-3y^2-x^2\right)=2xy-3x^2-y^2$	1

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	a)	$\frac{dy}{dx}=\frac{2xy-3x^2-y^2}{2xy-3y^2-x^2}$	1
		---	
	b)	Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ if $x=a\cos^3\theta, y=b\sin^3\theta$	**04**
	Ans	$x=a\cos^3\theta$	
		$\therefore \frac{dx}{d\theta}=3a\cos^2\theta\left(-\sin\theta\right)$	1
		$=-3a\cos^2\theta\sin\theta$	
		$y=b\sin^3\theta$	
		$\therefore \frac{dy}{d\theta}=3b\sin^2\theta\cos\theta$	1
		$=3b\sin^2\theta\cos\theta$	
		$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$=\frac{3b\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$	1
		$=-\frac{b}{a}\tan\theta$	
		$at\ \theta=\frac{\pi}{4}$	
		$\frac{dy}{dx}=-\frac{b}{a}\tan\frac{\pi}{4}$	
		$=-\frac{b}{a}$	1
		---	
	c)	A manufacture can sell x items per week at price $\left(23-0.001x\right)$ rupees each. It cost $\left(5x+2000\right)$ rupees to produce x items Find the number items to be produced eper week for maximum profit.	**04**
	Ans	Let number of item be x	
		Selling price $=\left(23-0.001x\right)x$	
		$=23x-0.001x^2$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	Cost price $=\left(5x+2000\right)$	
		profit $=$ selling price $-$ cost price	
		$\therefore\ p=23x-0.001x^{2}-\left(5x+2000\right)$	½
		$=23x-0.001x^{2}-5x-2000$	
		$=18x-0.001x^{2}-2000$	½
		$\frac{dp}{dx}=18-0.002x$	½
		$\frac{d^{2}p}{dx^{2}}=-0.002$	½
		$\therefore$ profit is maximum	
		Let	
		$18-0.002x=0$	½
		$x=\frac{18}{0.002}=9000$	1
		--	
	d)	Find the radius of curvature of the curve $y=e^{x}$ at the point where it crosses the Y-axis.	**04**
	Ans	$y=e^{x}$	
		$\frac{dy}{dx}=e^{x}$	1
		$\frac{d^{2}y}{dx^{2}}=e^{x}$	1
		curve crosses Y-axis $\therefore x=0$	
		$\frac{dy}{dx}=e^{0}=1$	½
		$\frac{d^{2}y}{dx^{2}}=e^{0}=1$	½
		$\rho=\frac{\left(1+\left(\frac{dy}{dx}\right)^{2}\right)^{\frac{3}{2}}}{\frac{d^{2}y}{dx^{2}}}$	
		$=\frac{\left(1+1^{2}\right)^{\frac{3}{2}}}{1}=2^{\frac{3}{2}}=2.828$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.		**Solve any THREE of the following**	**12**
	a)	Find equation of tangent and normal to the curve $2x^2-xy+3y^2=18$ at $(3,1)$	**04**
	Ans	$2x^2-xy+3y^2=18$	
		$\therefore 4x-\left(x\frac{dy}{dx}+y.1\right)+6y\frac{dy}{dx}=0$	1
		$\therefore 4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$	
		$\therefore (6y-x)\frac{dy}{dx}=y-4x$	½
		$\therefore \frac{dy}{dx}=\frac{y-4x}{6y-x}$	½
		at $(3,1)$	
		$\therefore \frac{dy}{dx}=\frac{1-4(3)}{6(1)-3}$	
		$\therefore \frac{dy}{dx}=\frac{-11}{3}$	
		$\therefore$ slope of tangent , $m=\frac{-11}{3}$	½
		Equation of tangent at $(3,1)$ is	
		$y-1=\frac{-11}{3}(x-3)$	
		$\therefore 3y-3=-11x+33$	
		$\therefore 11x+3y-36=0$	½
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{3}{11}$	½
		Equation of normal at $(3,1)$ is	
		$y-1=\frac{3}{11}(x-3)$	
		$\therefore 11y-11=3x-9$	
		$\therefore 3x-11y+2=0$	½
		--	
	b)	Differentiate with respect to x : $x^x+5^x+x^5+5^5$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)Ans	$y = x^x + 5^x + x^5 + 5^5$	
		Let $u = x^x$	½
		$\log u = \log x^x$	
		$\log u = x \log x$	1
		$\frac{1}{u}\frac{du}{dx} = x.\frac{1}{x} + \log x.1$	½
		$\frac{du}{dx} = u\left(1+\log x\right) = x^x\left(1+\log x\right)$	
		$\therefore \frac{dy}{dx} = x^x\left(1+\log x\right) + 5^x \log 5 + 5x^4$	2
		---	
	c)	If $x^3.y^2 = \left(x+y\right)^5$, show that $\frac{dy}{dx} = \frac{y}{x}$	**04**
	Ans	$x^3.y^2 = \left(x+y\right)^5$	
		$\log\left(x^3.y^2\right) = \log\left(x+y\right)^5$	½
		$\log x^3 + \log y^2 = 5\log\left(x+y\right)$	½
		$3\log x + 2\log y = 5\log\left(x+y\right)$	½
		$3\frac{1}{x} + 2\frac{1}{y}\frac{dy}{dx} = 5\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$	1
		$\frac{3}{x} + \frac{2}{y}\frac{dy}{dx} = \frac{5}{x+y} + \frac{5}{x+y}\frac{dy}{dx}$	
		$\frac{2}{y}\frac{dy}{dx} - \frac{5}{x+y}\frac{dy}{dx} = \frac{5}{x+y} - \frac{3}{x}$	
		$\frac{dy}{dx}\left(\frac{2}{y} - \frac{5}{x+y}\right) = \frac{5x-3x-3y}{x\left(x+y\right)}$	
		$\frac{dy}{dx}\left(\frac{2x+2y-5y}{y\left(x+y\right)}\right) = \frac{5x-3x-3y}{x\left(x+y\right)}$	½
		$\frac{dy}{dx}\left(\frac{2x-3y}{y}\right) = \frac{2x-3y}{x}$	
		$\frac{dy}{dx} = \frac{y}{x}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate: $\int \frac{e^x\left(x+1\right)}{\sin^2\left(xe^x\right)}dx.$	**04**
	Ans	$\int \frac{e^x\left(x+1\right)}{\sin^2\left(xe^x\right)}dx$	
		put $xe^x = t$	
		$\left(xe^x + e^x.1\right)dx = dt$	
		$e^x\left(x+1\right)dx = dt$	2
		$= \int \frac{dt}{\sin^2 t}$	
		$= \int \cos ec^2 t dt$	½
		$= -\cot t + c$	1
		$= -\cot\left(xe^x\right) + c$	½
4		**Solve any <u>THREE</u> of the following:**	**04**
	a)	Evaluate: $\int \frac{x-3}{x^3-3x^2-16x+48}dx$	
		$\int \frac{x-3}{x^3-3x^2-16x+48}dx$	
		$= \int \frac{x-3}{\left(x-3\right)\left(x-4\right)\left(x+4\right)}dx$	½
		$= \int \frac{dx}{\left(x-4\right)\left(x+4\right)}$	
		Consider $\frac{1}{\left(x-4\right)\left(x+4\right)} = \frac{A}{x-4} + \frac{B}{x+4}$	½
		$1 = A\left(x+4\right) + B\left(x-4\right)$	
		put $x = 4$ $A = \frac{1}{8},$	½
		put $x = -4$ $B = -\frac{1}{8}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$\therefore\int\frac{dx}{(x-4)(x+4)}$	
		$=\int\left(\frac{\frac{1}{8}}{x-4}+\frac{-\frac{1}{8}}{x+4}\right)dx$	
		$=\frac{1}{8}\left(\log(x-4)-\log(x+4)\right)+c$	2
	b)	Evaluate : $\int\frac{1}{2+3\cos x}dx$	**04**
	Ans	$\int\frac{1}{2+3\cos x}dx$	
		Put $\tan\frac{x}{2}=t \quad \cos x=\frac{1-t^2}{1+t^2}, dx=\frac{2dt}{1+t^2}$	1
		$\therefore\int\frac{dx}{2+3\cos x}=\int\frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$	
		$=2\int\frac{1}{5-t^2}dt$	1
		$=2\int\frac{1}{\left(\sqrt{5}\right)^2-t^2}dt$	½
		$=2\times\frac{1}{2\sqrt{5}}\log\left(\frac{\sqrt{5}+t}{\sqrt{5}-t}\right)+c$	1
		$=\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right)+c$	½
	c)	Evalute: $\int e^x.\sin 4xdx$	**04**
	Ans	$\int e^x.\sin 4xdx$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$=\sin 4x\int e^{x}dx-\int\left(\int e^{x}dx\frac{d}{dx}\sin 4x\right).dx$	1
		$=\sin 4xe^{x}-\int\cos 4x\cdot 4\cdot e^{x}dx$	½
		$=\sin 4xe^{x}-4\left[\cos 4x\int e^{x}dx-\int\left(\int e^{x}dx\cdot\frac{d}{dx}\cos 4x\right)dx\right]$	1
		$=\sin 4xe^{x}-4\left[\cos 4xe^{x}-\int\left(-\sin 4x\cdot 4\cdot e^{x}\right)dx\right]$	½
		$=\sin 4xe^{x}-4\left[\cos 4xe^{x}+4\int\sin 4x\cdot e^{x}dx\right]$	
		$=\sin 4xe^{x}-4\cos 4xe^{x}-16I$	
		$I+16I=\sin 4xe^{x}-4\cos 4xe^{x}$	½
		$17I=\sin 4xe^{x}-4\cos 4xe^{x}$	
		$I=\frac{1}{17}\left(\sin 4xe^{x}-4\cos 4xe^{x}\right)$	½
		--	
	d)	Evaluate: $\int\frac{e^{x}}{\left(e^{x}-1\right)\left(e^{x}+1\right)}dx$	**04**
	Ans	$\int\frac{e^{x}}{\left(e^{x}-1\right)\left(e^{x}+1\right)}dx$	
		put $e^{x}=t$	
		$e^{x}dx=dt$	1
		$\int\frac{e^{x}}{\left(e^{x}-1\right)\left(e^{x}+1\right)}dx=\int\frac{dt}{\left(t-1\right)\left(t+1\right)}$	
		consider $\frac{1}{\left(t-1\right)\left(t+1\right)}=\frac{A}{t-1}+\frac{B}{t+1}$	½
		$1=A\left(t+1\right)+B\left(t-1\right)$	
		put $t=1,A=\frac{1}{2}$	½
		put $t=-1,B=-\frac{1}{2}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$\frac{1}{(t-1)(t+1)}=\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}$	
		$\int\frac{dt}{(t-1)(t+1)}=\int\left(\frac{\frac{1}{2}}{t-1}+\frac{-\frac{1}{2}}{t+1}\right)dt$	
		$=\frac{1}{2}\log(t-1)-\frac{1}{2}\log(t+1)+c$	1
		$=\frac{1}{2}\log\left(e^{x}-1\right)-\frac{1}{2}\log\left(e^{x}+1\right)+c$	½
		--	
	e)	Evaluate : $\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx$	**04**
	Ans	$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx$	
		$=\int_0^{\frac{\pi}{2}}\frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}dx$	½
		$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx---------(1)$	
		by property	
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$	1
		$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx---------(2)$	1
		add (1) and (2)	
		$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$2I=\int_0^{\frac{\pi}{2}}1dx$	
		$\therefore 2I=\left[x\right]_0^{\frac{\pi}{2}}$	½
		$=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		OR	
		$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan x}}dx--------(1)$	
		by property	
		$\text{I}=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}dx$	1
		$I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\cot x}}dx$	1
		$\therefore I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\frac{1}{\tan x}}}dx$	½
		$=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}dx---------(2)$	
		$add\ (1)$ and (2)	
		$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}+1}{\sqrt{\tan x}+1}dx$	½
		$2I=\int_0^{\frac{\pi}{2}}1dx$	
		$\therefore 2I=\left[x\right]_0^{\frac{\pi}{2}}$	½
		$=\frac{\pi}{2}-0$	
		$I=\frac{\pi}{4}$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.		**Solve any TWO of the following:**	**12**
	a)	Find the area bounded by two parabolas $y^2 = 2x$ and $x^2 = 2y$.	**06**
	Ans	$y^2 = 2x$ and $x^2 = 2y$ put $y = \frac{x^2}{2}$ in $y^2 = 2x$ $\therefore \left(\frac{x^2}{2}\right)^2 = 2x$ $x^4 - 8x = 0$ $x\left(x^3 - 2^3\right) = 0$ $x = 0, x = 2$ $Let \quad y_1 = \sqrt{2x},$ $y_2 = \frac{x^2}{2}$	2
		$\text{Area} = \int_a^b \left(y_2 - y_1\right) dx$ $= \int_0^2 \left(\frac{x^2}{2} - \sqrt{2x}\right) dx$	1
		$= \int_0^2 \left(\frac{x^2}{2} - \sqrt{2}x^{\frac{1}{2}}\right) dx$ $= \left[\frac{x^3}{6} - \frac{\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2$	1
		$= \frac{2^3}{6} - \frac{2}{3} \times \sqrt{2} \times 2^{\frac{3}{2}} - 0$	1
		$= \frac{4}{3} = 1.333$	1
		--	
5.		**Solve the following:**	**06**
	b) (i)	Form the differential equation from the relation, $y = A.e^x + B.e^{-x}$	**03**

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	Ans	$y = A.e^{x} + B.e^{-x}$	
		$\therefore \frac{dy}{dx} = A.e^{x} - B.e^{-x}$	1
		$\therefore \frac{d^2y}{dx^2} = A.e^{x} + B.e^{-x}$	1
		$\therefore \frac{d^2y}{dx^2} = y$	
		$\therefore \frac{d^2y}{dx^2} - y = 0$	1
		--	
	(ii)	Solve $\frac{dy}{dx} + y\cot x = \cos ecx$	**03**
	Ans	$\frac{dy}{dx} + y\cot x = \cos ecx$	
		$\therefore$ Comparing with $\frac{dy}{dx} + Py = Q$	
		$P = \cot x \ , \ Q = \cos ecx$	
		Integrating factor $IF = e^{\int \cot x dx}$ $= e^{\log(\sin x)}$ $= \sin x$	1
		$\therefore \ y.IF = \int Q.IF dx + c$	
		$\therefore y\sin x = \int \cos ecx.\sin x \ dx$	1
		$\therefore y\sin x = \int 1 \ dx$	
		$\therefore y\sin x = x + c$	1
		--	
	c)	The velocity of a particle is given by $\frac{dx}{dt} = 3t^2 - 6t + 8$. Find distance covered in 2 seconds given that $x = 0$ at $t = 0$	**06**
	Ans	$\frac{dx}{dt} = 3t^2 - 6t + 8$	
		$\therefore dx = \left(3t^2 - 6t + 8\right)dt$	
		$\therefore \int dx = \int \left(3t^2 - 6t + 8\right)dt$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$\therefore x=\frac{3t^3}{3}-\frac{6t^2}{2}+8t+c$	2
		$\therefore x=t^3-3t^2+8t+c$	
		given that $x=0$ at $t=0$	
		$\therefore c=0$	1
		$\therefore x=t^3-3t^2+8t$	1
		Distance covered in 2 sec,	
		$\therefore x=(2)^3-3(2)^2+8(2)$	
		$\therefore x=12$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)(i)	Solve the following system of equations by Jacobi-Iteration method (Two iterations)	**03**
		$15x+2y+z=18$,	
		$2x+20y-3z=19$,	
		$3x-6y+25z=22$	
	Ans	$15x+2y+z=18$,	
		$2x+20y-3z=19$,	
		$3x-6y+25z=22$	
		$x=\frac{1}{15}(18-2y-z)$	
		$y=\frac{1}{20}(19-2x+3z)$	1
		$z=\frac{1}{25}(22-3x+6y)$	
		Starting with $x_0=y_0=z_0=0$	
		$x_1=1.2$	
		$y_1=0.95$	1
		$z_1=0.88$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$x_2 = 1.015$ $y_2 = 0.962$ $z_2 = 0.964$	1
	a)(ii)	Solve the following system of equations by using Gauss-Seidal method $\left(\text{Two iterations} \right)$ $5x - 2y + 3z = 18;$ $x + 7y - 3z = 22$, $2x - y + 6z = 22$	**03**
	Ans	$5x - 2y + 3z = 18;$ $x + 7y - 3z = 22$, $2x - y + 6z = 22$ $x = \frac{1}{5}\left(18 + 2y - 3z\right)$ $y = \frac{1}{7}\left(22 - x + 3z\right)$ $z = \frac{1}{6}\left(22 - 2x + y\right)$	1
		Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 3.6$ $y_1 = 2.629$ $z_1 = 2.905$	1
		$x_2 = 2.909$ $y_2 = 3.972$ $z_2 = 3.359$	1
	b)	Solve the following equations by Gauss elimination method. $6x - y - z = 19$ $3x + 4y + z = 26$ $x + 2y + 6z = 22$	**06**

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)Ans	$6x-y-z=19$ $\quad\quad$ $36x-6y-6z=114$	
		$3x+4y+z=26$ $\quad$ and $\quad$ $x+2y+6z=22$	
		$+$ ______ $\quad\quad$ $+$ ______	
		$9x+3y=45$ $\quad\quad$ $37x-4y=136$	
		$3x+y=15$ $\quad\quad$ $37x-4y=136$	1+1
		$12x+4y=60$	
		$37x-4y=136$	
		$+$ ______	1
		$49x=196$	
		$\therefore x=4$	1
		$y=3$	1
		$z=2$	1
		Note: *In the above solution, first x is eliminated and then z is eliminated to find the value of y first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.*	
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	c)	Using Newton-Raphson method to find the approximate value of $\sqrt[3]{100}$ (perform 4 iterations)	**06**
	Ans	Let $x=\sqrt[3]{100}$	½
		$\therefore x^3-100=0$	
		$f(x)=x^3-100$	
		$f(4)=-36<0$	½
		$f(5)=25>0$	
		$f'(x)=3x^2$	½
		Initial root $x_0=5$	½
		$\therefore f'(5)=75$	
		$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=5-\dfrac{f(5)}{f'(5)}=4.6667$	1
		$x_2=4.6667-\dfrac{f(4.6667)}{f'(4.6667)}=4.6417$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$x_3 = 4.6417 - \frac{f(4.6417)}{f^{'}(4.6417)} = 4.6416$	1
		$x_4 = 4.6416 - \frac{f(4.6416)}{f^{'}(4.6416)} = 4.6416$	1
		OR	
		$Let\ f(x) = x^3 - 100$	½
		$f(4) = -36 < 0 \qquad f(5) = 25 > 0$	½
		$f'(x) = 3x^2$	½
		Initial root x_0=5	½
		$x_i = x - \frac{f(x)}{f^{'}(x)} = x - \frac{x^3 - 100}{3x^2}$	
		$= \frac{3x^3 - x^3 + 100}{3x^2}$	
		$= \frac{2x^3 + 100}{3x^2}$	2
		$x_1 = 4.6667$	½
		$x_2 = 4.6417$	½
		$x_3 = 4.6416$	½
		$x_4 = 4.6416$	½
		--- **_Important Note_** *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* ---	