Nonlinear Op-Amp Circuits

- Most typical applications require op amp and its components to act linearly
  - I-V characteristics of passive devices such as resistors, capacitors should be described by linear equation (Ohm's Law)
  - For op amp, linear operation means input and output voltages are related by a constant proportionality (A_v should be constant)
- Some application require op amps to behave in nonlinear manner (logarithmic and antilogarithmic amplifiers)

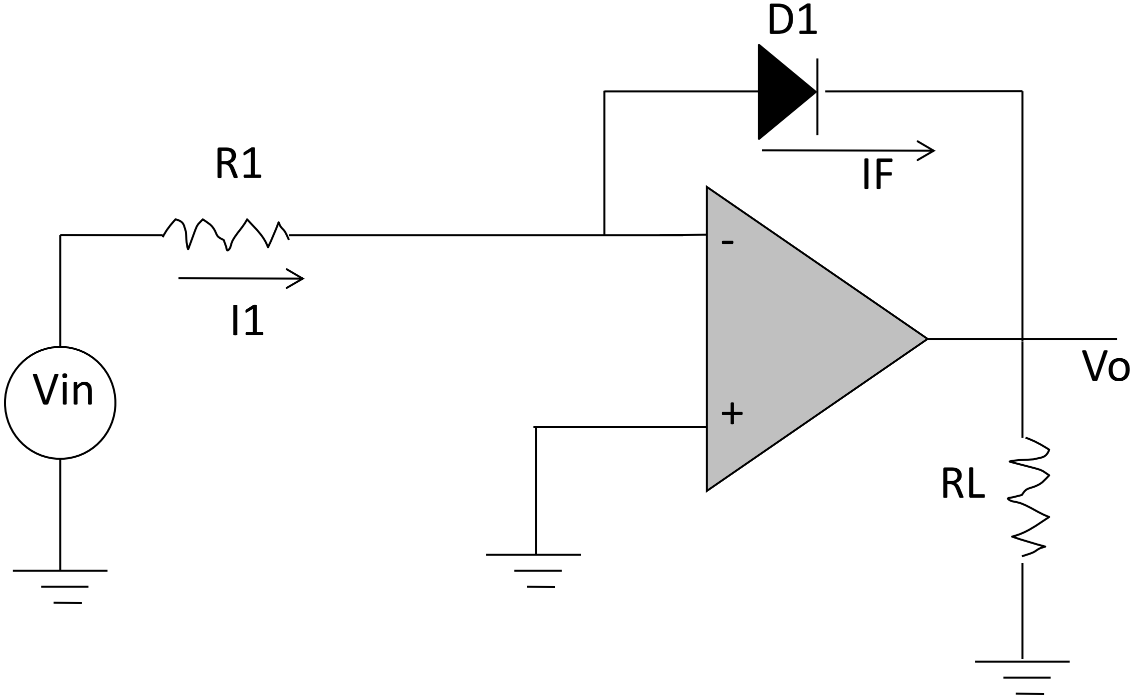
Logarithmic Amplifier

- Output voltage is proportional to the logarithm of input voltage
- A device that behaves nonlinearly (logarithmically) should be used to control gain of op amp
  - Semiconductor diode
- Forward transfer characteristics of silicon diodes are closely described by Shockley's equation

  $I_F = I_s e^{(V_F/\eta V_T)}$

  - I_s is diode saturation (leakage) current
  - e is base of natural logarithms (e = 2.71828)
  - V_F is forward voltage drop across diode
  - V_T is thermal equivalent voltage for diode (26 mV at 20°C)
  - η is emission coefficient or ideality factor (2 for currents of same magnitude as I_S to 1 for higher values of I_F)

Basic Log Amp operation

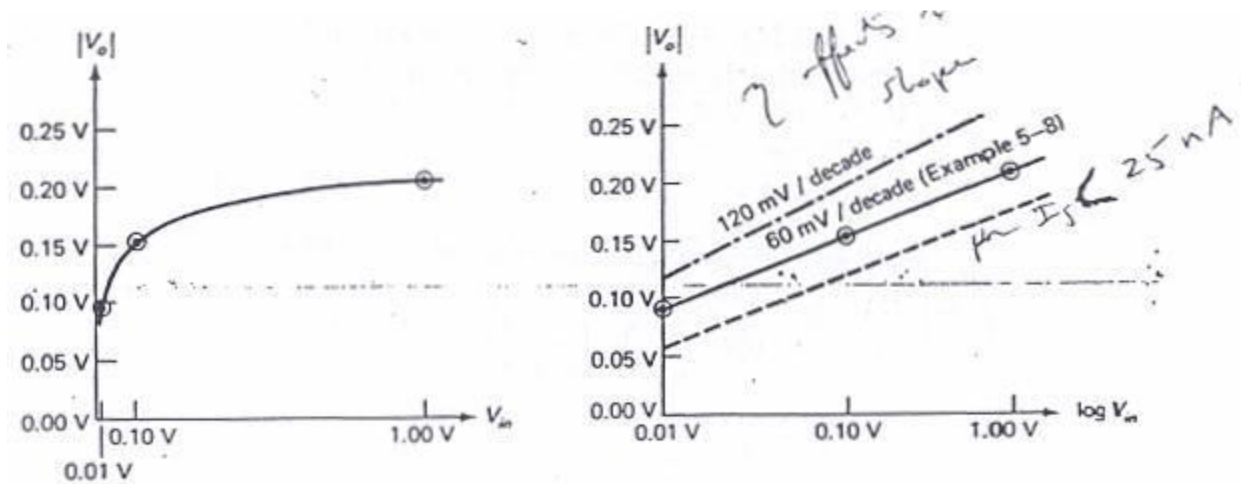


- $I1 = Vin/R1$
- $IF = - I1$
- $IF = - Vin/R1$
- $V0 = -VF = -\eta VT \ln(IF/IS)$
- $V0 = -\eta VT \ln[Vin/(R1IS)]$
- $rD = 26\ mV / IF$
- $IF < 1\ mA$ (log amps)

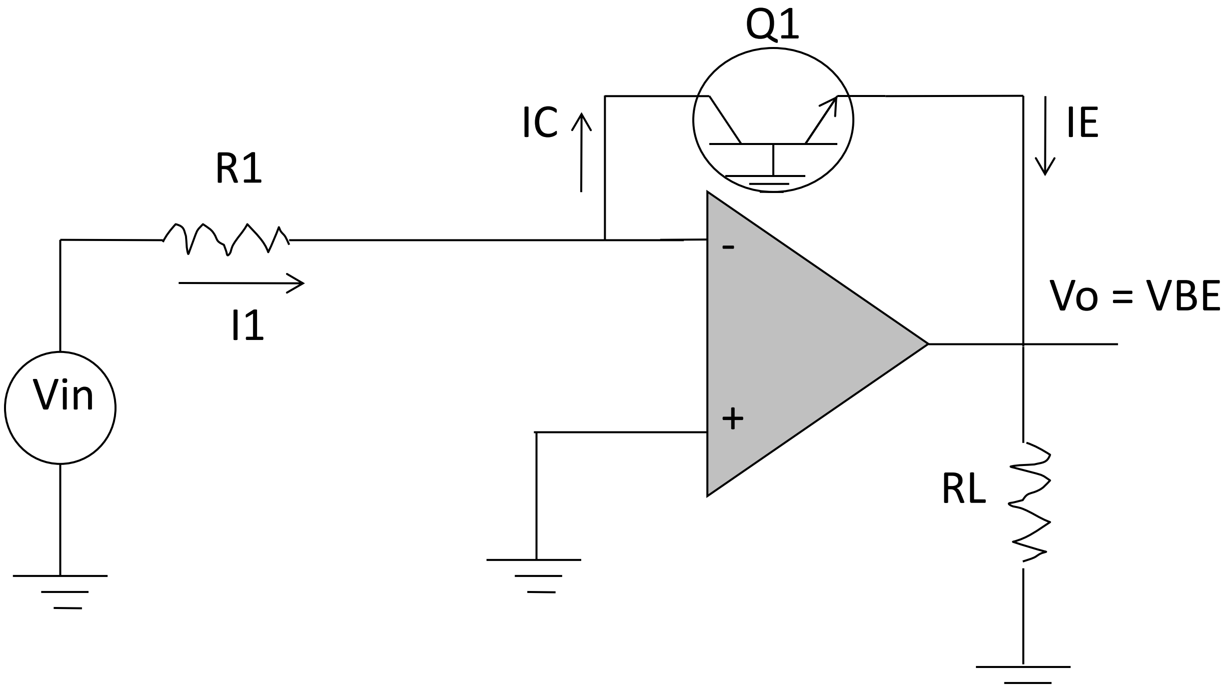
- At higher current levels ($IF > 1\ mA$) diodes begin to behave somewhat linearly

Logarithmic Amplifier

- Linear graph: voltage gain is very high for low input voltages and very low for high input voltages
- Semilogarithmic graph: straight line proves logarithmic nature of amplifier's transfer characteristic
- Transfer characteristics of log amps are usually expressed in terms of slope of V_0 versus V_{in} plot in milivolts per decode
- η affects slope of transfer curve; I_S determines the y intercept



Additional Log Amp Variations



$$IC = IESe^{(V_{BE}/V_T)}$$

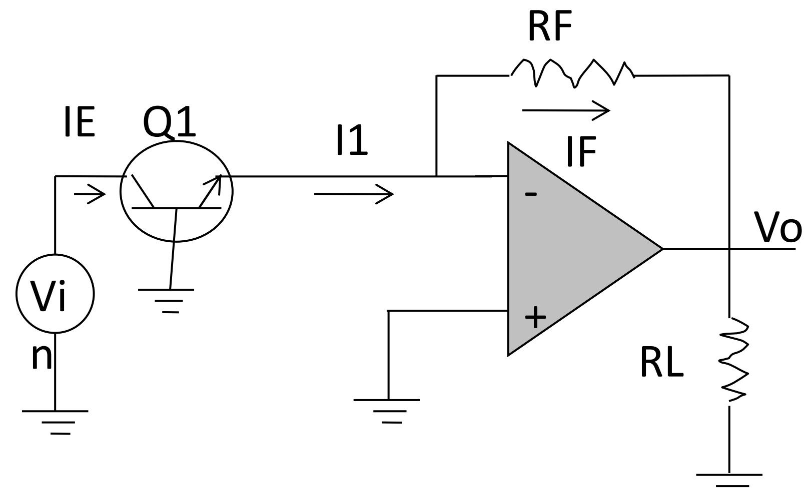
- IES is emitter saturation current
- VBE is drop across base-emitter junction

- Often a transistor is used as logging element in log amp (transdiode configuration)
- Transistor logging elements allow operation of log amp over wider current ranges (greater dynamic range)

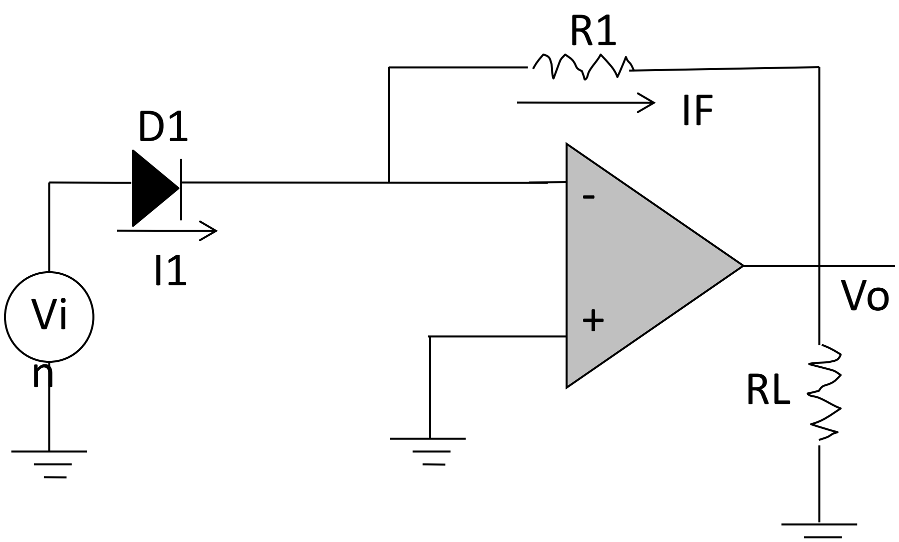
Antilogarithmic Amplifier

- Output of an antilog amp is proportional to the antilog of the input voltage
- with diode logging element
  - $V_0 = -R_F I_S e^{(V_{in}/V_T)}$
- With transdiode logging element
  - $V_0 = -R_F I_{ES} e^{(V_{in}/V_T)}$
- As with log amp, it is necessary to know saturation currents and to tightly control junction temperature

Antilogarithmic Amplifier



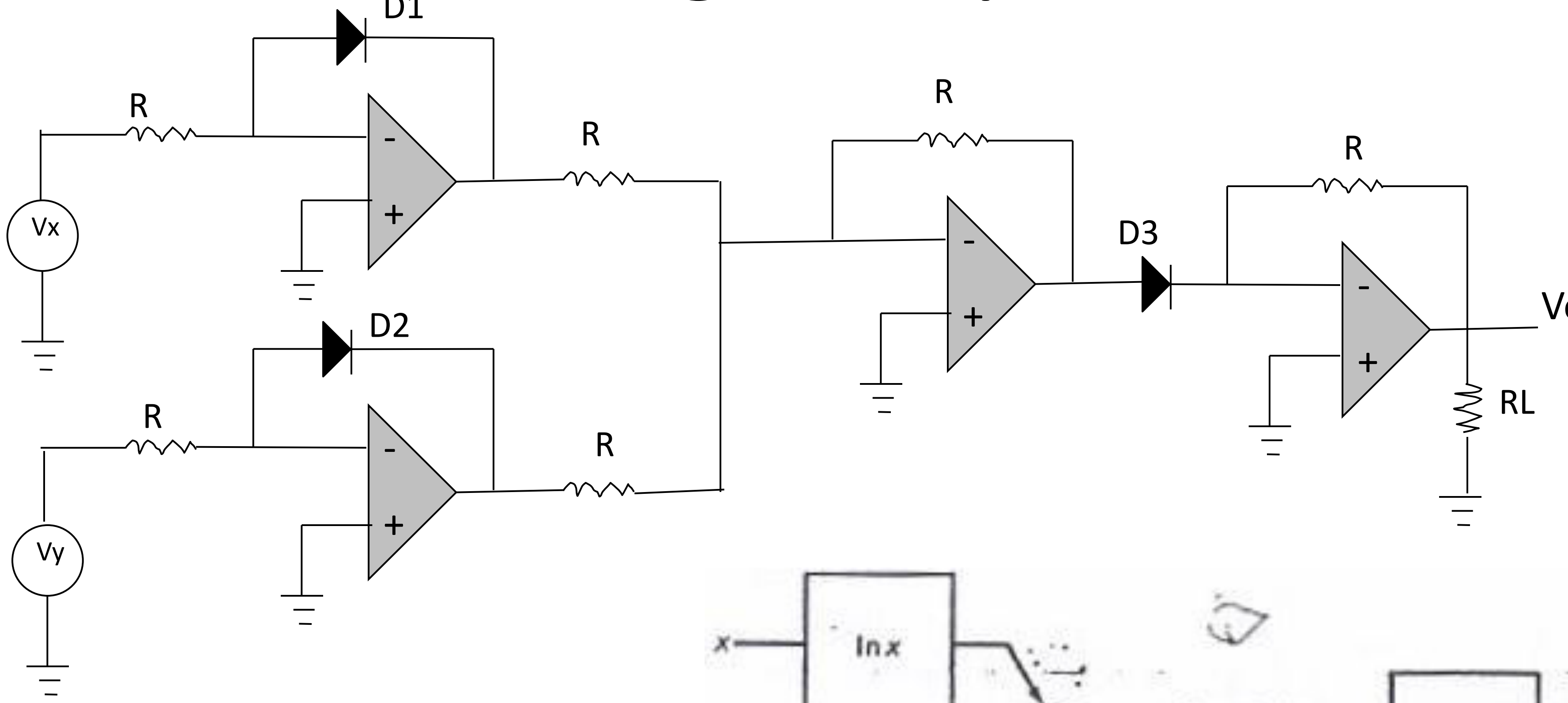
$(\alpha = 1)\ I1 = IC = IE$

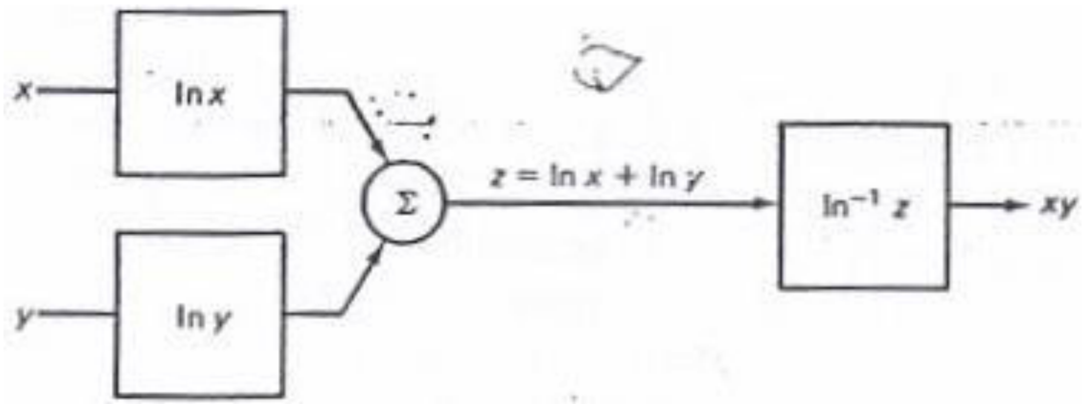


Logarithmic Amplifier Applications

- Logarithmic amplifiers are used in several areas
  - Log and antilog amps to form analog multipliers
  - Analog signal processing
- Analog Multipliers
  - $\ln xy = \ln x + \ln y$
  - $\ln (x/y) = \ln x - \ln y$

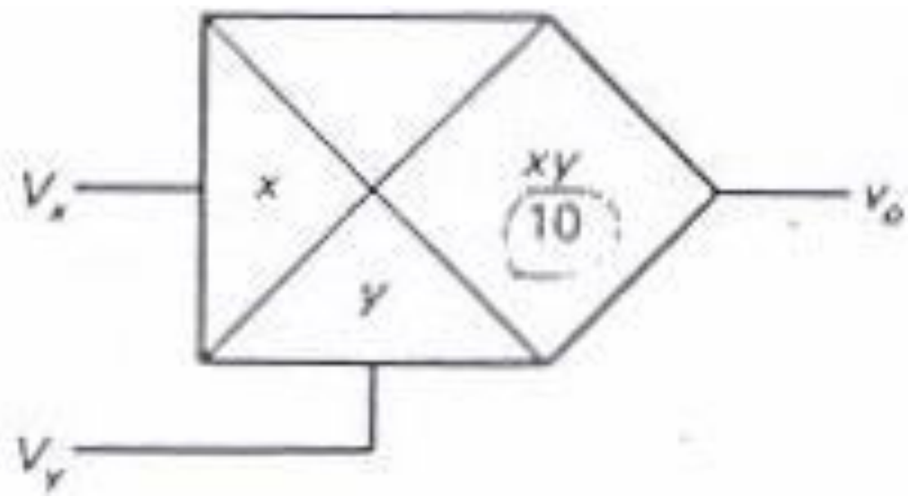
Analog Multipliers





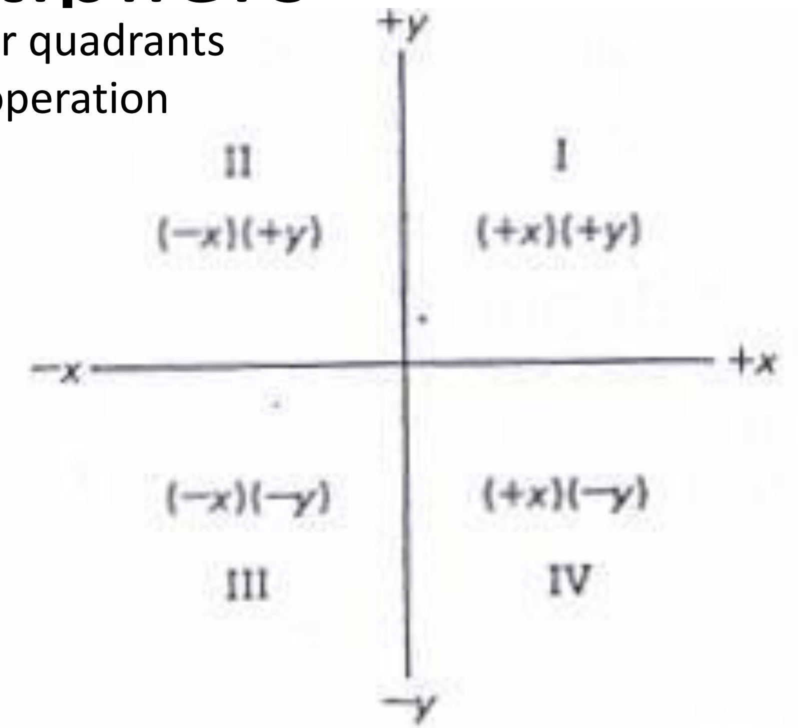
One-quadrant multiplier: inputs must both be of same polarity

Analog Multipliers



General symbol

Four quadrants of operation

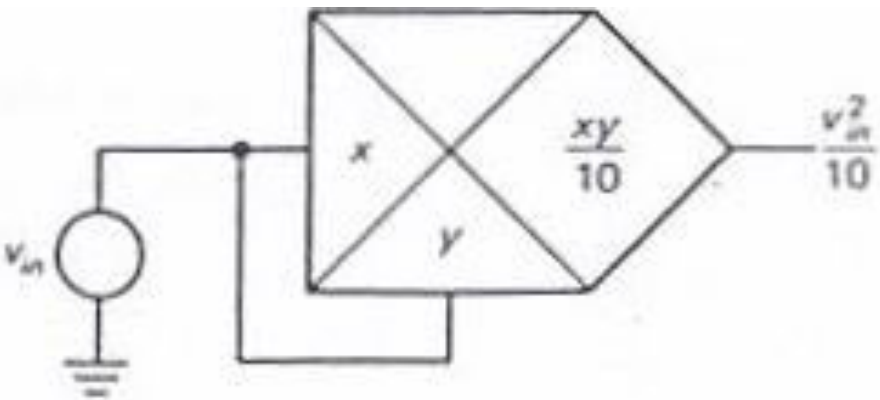


Two-quadrant multiplier: one input should have positive voltages, other input could have positive or negative voltages

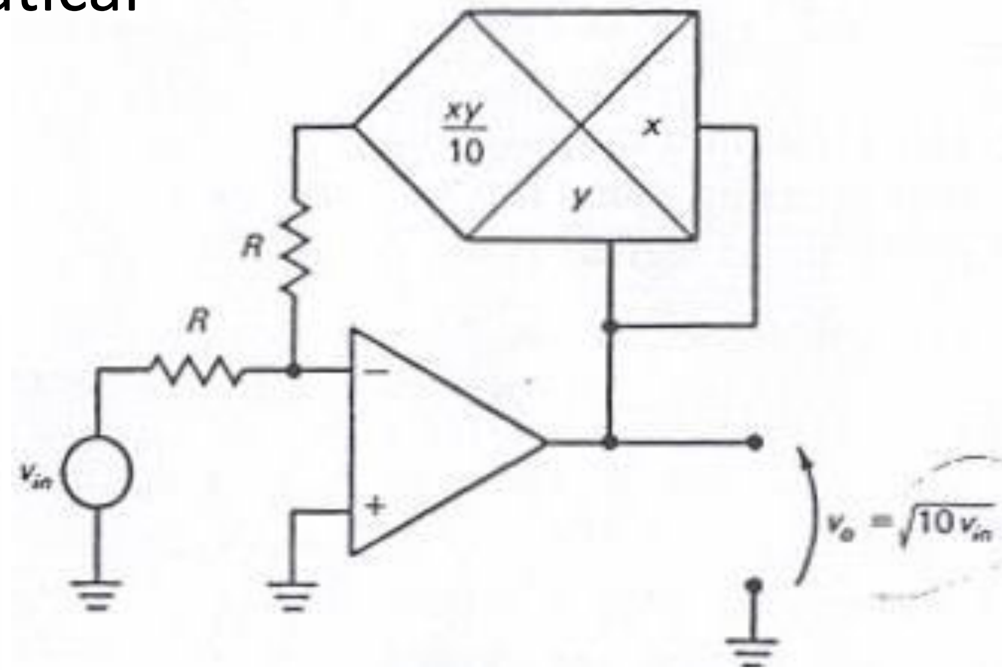
Four-quadrant multiplier: any combinations of polarities on their inputs

Analog Multipliers

Implementation of mathematical operations



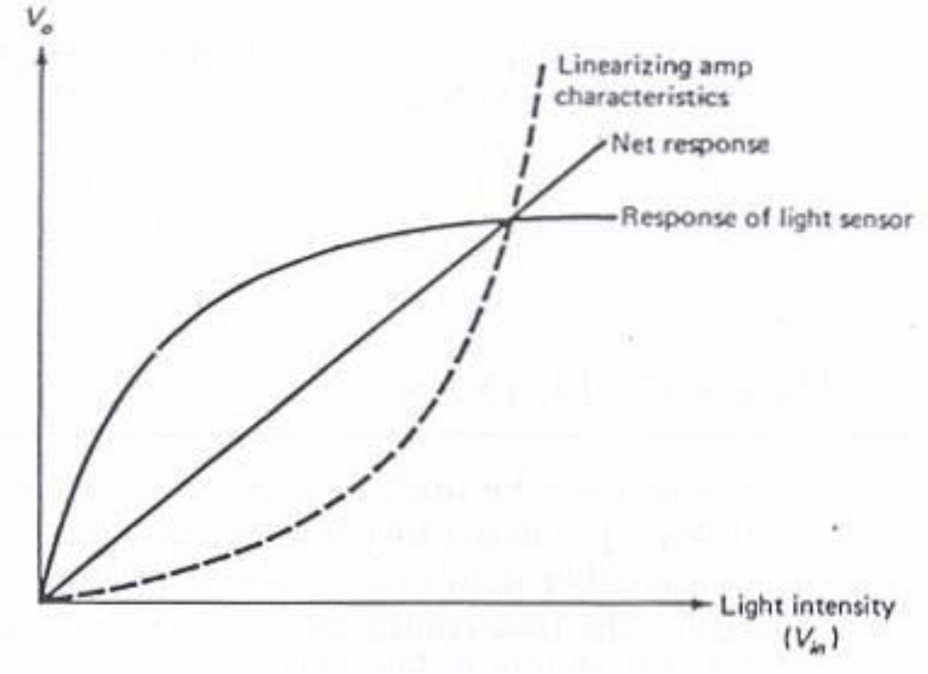
Squaring Circuit



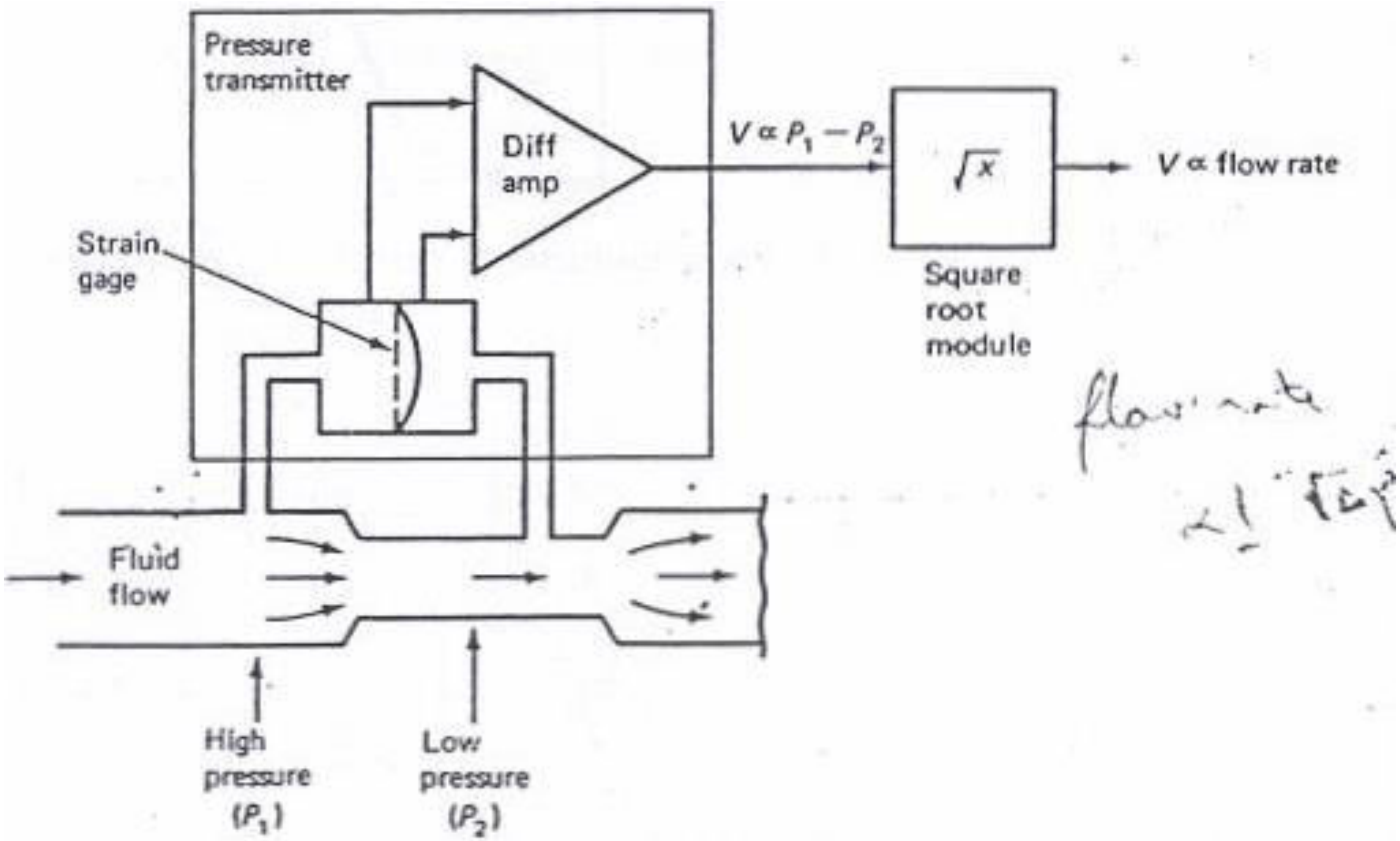
Square root Circuit

Signal Processing

- Many transducers produce output voltages that vary nonlinearly with physical quantity being measured (thermistor)
- Often It is desirable to linearize outputs of such devices; logarithmic amps and analog multipliers can be used for such purposes
- Linearization of a signal using circuit with complementary transfer characteristics



Pressure Transmitter



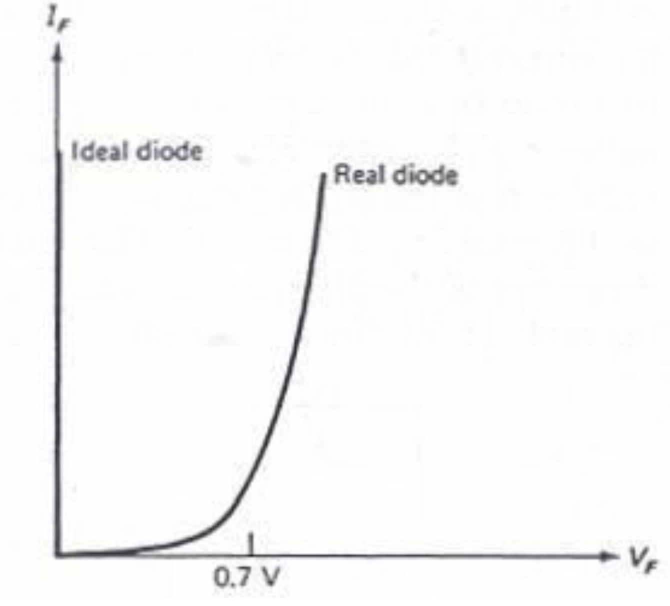
Pressure transmitter produces an output voltage proportional to difference in pressure between two sides of a strain gage sensor

Pressure Transmitter

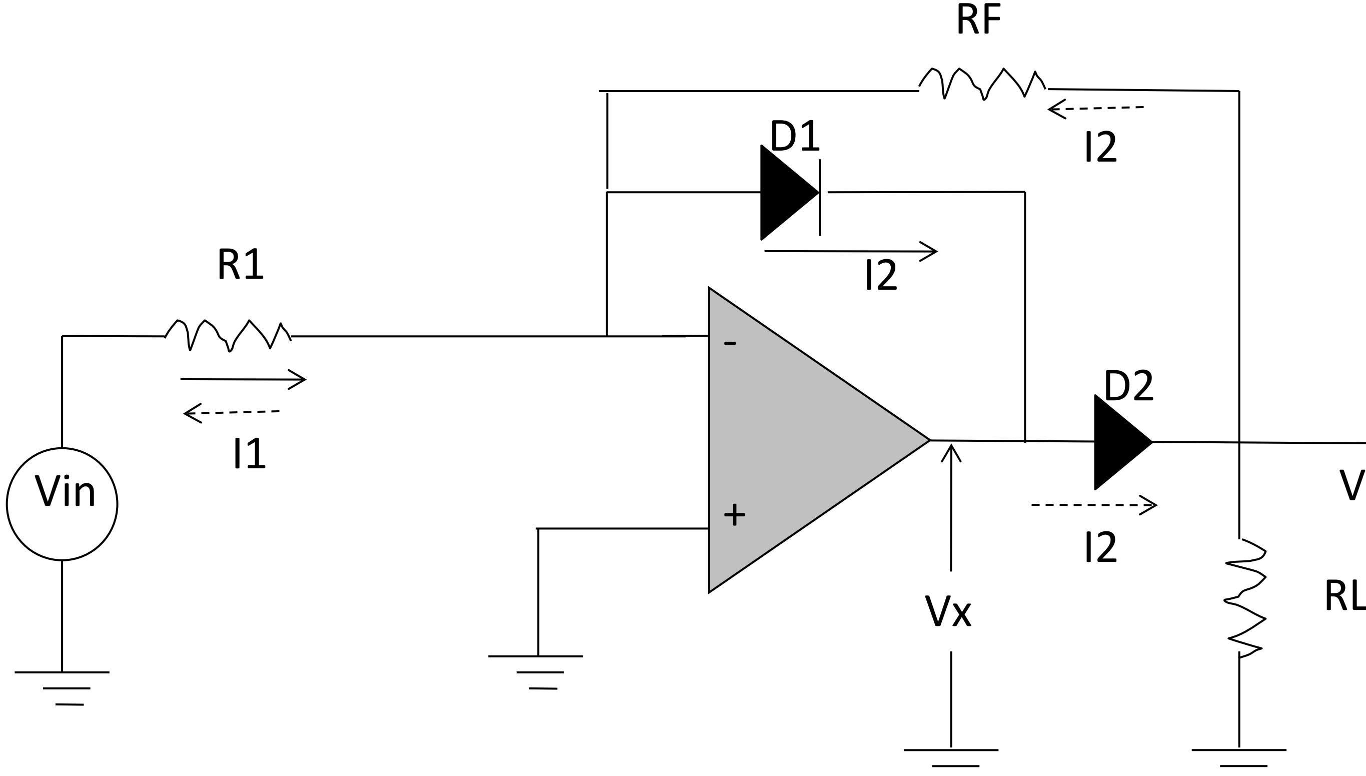
- A venturi is used to create pressure differential across strain gage
- Output of transmitter is proportional to pressure differential
- Fluid flow through pipe is proportional to square root of pressure differential detected by strain gage
- If output of transmitter is processed through a square root amplifier, an output directly proportional to flow rate is obtained

Precision Rectifiers

- Op amps can be used to form nearly ideal rectifiers (convert ac to dc)
- Idea is to use negative feedback to make op amp behave like a rectifier with near-zero barrier potential and with linear I/O characteristic
- Transconductance curves for typical silicon diode and an ideal diode

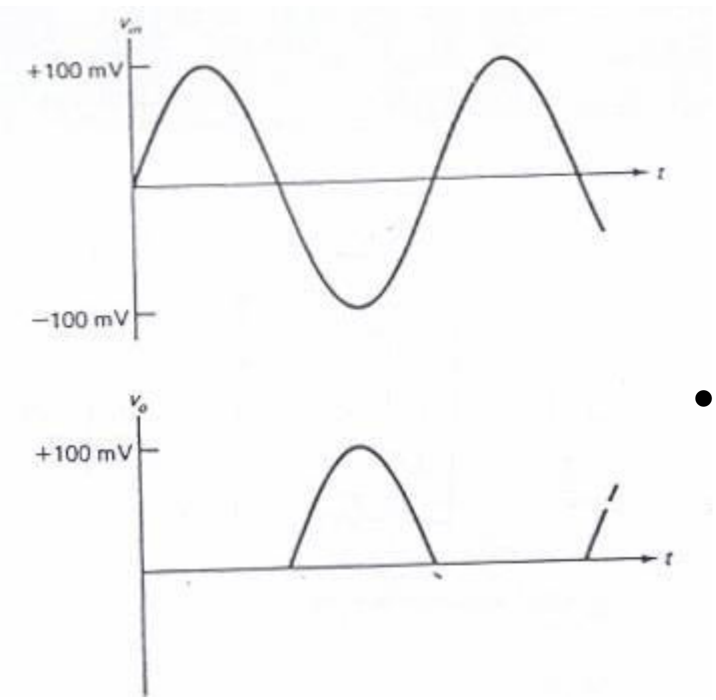


Precision Half-Wave Rectifier



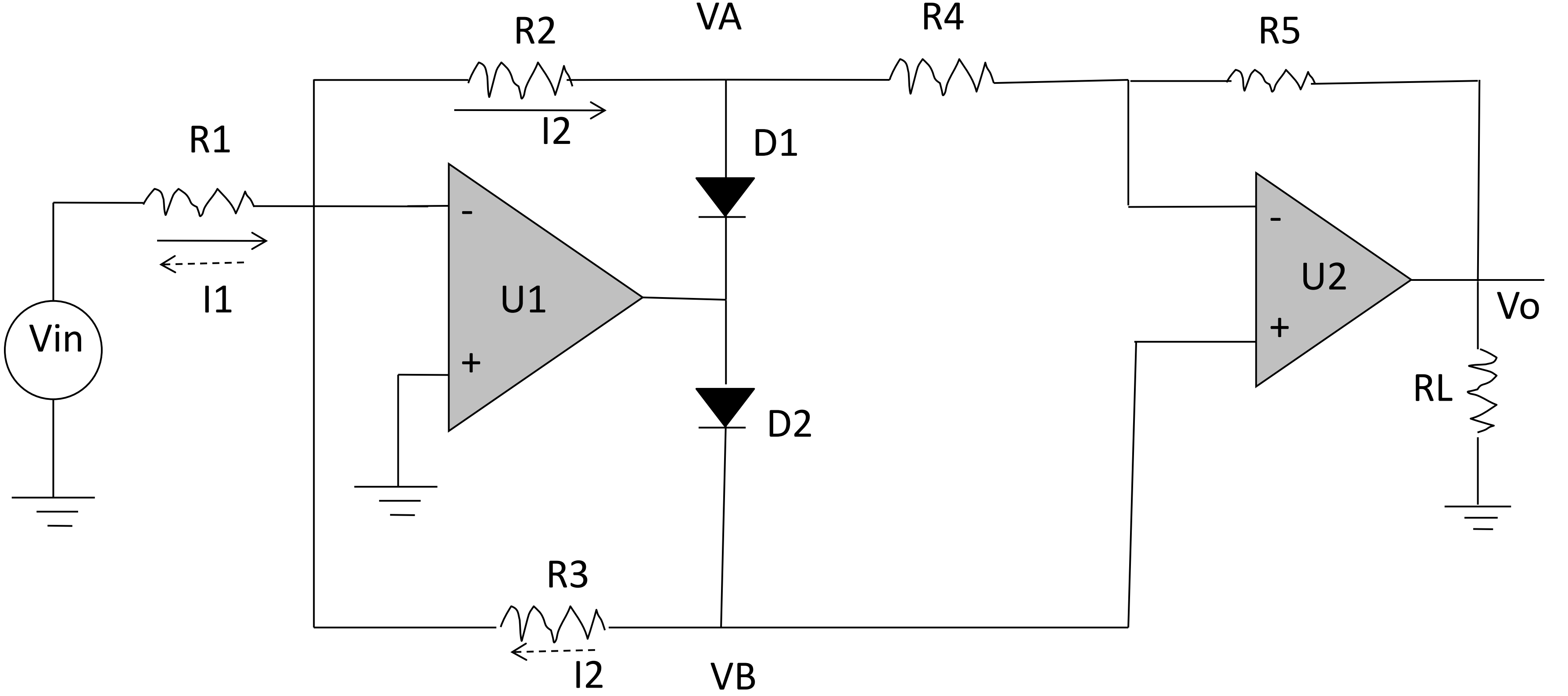
- Solid arrows represent current flow for positive half-cycles of V_{in} and dashed arrows represent current flow for negative half-cycles

Precision Half-Wave Rectifier



- If signal source is going positive, output of op amp begins to go negative, forward biasing D_1
  - Since D_1 is forward biased, output of op amp V_x will reach a maximum level of ~ -0.7V regardless of how far positive V_{in} goes
  - This is insufficient to appreciably forward bias D_2, and V_0 remains at 0V
- On negative-going half-cycles, D_1 is reverse-biased and D_2 is forward biased
  - Negative feedback reduces barrier potential of D_2 to $0.7V/A_{OL}$ (~ = 0)
  - Gain of circuit to negative-going portions of V_{in} is given by $A_V = -R_F/R_1$

Precision Full-Wave Rectifier



- Solid arrows represent current flow for positive half-cycles of V_{in} and dashed arrows represent current flow for negative half-cycles

Precision Full-Wave Rectifier

- Positive half-cycle causes D_1 to become forward-biased, while reverse-biasing D_2
  - $V_B = 0$ V
  - $V_A = -V_{in} R_2/R_1$
  - Output of U_2 is $V_0 = -V_A R_5/R_4 = V_{in} (R_2R_5/R_1R_4)$
- Negative half-cycle causes U_1 output positive, forward-biasing D_2 and reverse-biasing D_1
  - $V_A = 0$ V
  - $V_B = -V_{in} R_3/R_1$
  - Output of U_2 (noninverting configuration) is $V_0 = V_B [1+ (R_5/R_4)]= - V_{in} [(R_3/R_1)+(R_3R_5/R_1R_4)$
  - if $R_3 = R_1/2$, both half-cycles will receive equal gain

Precision Rectifiers

- Useful when signal to be rectified is very low in amplitude and where good linearity is needed
- Frequency and power handling limitations of op amps limit the use of precision rectifiers to low-power applications (few hundred kHz)
- Precision full-wave rectifier is often referred to as absolute magnitude circuit

ACTIVE FILTERS

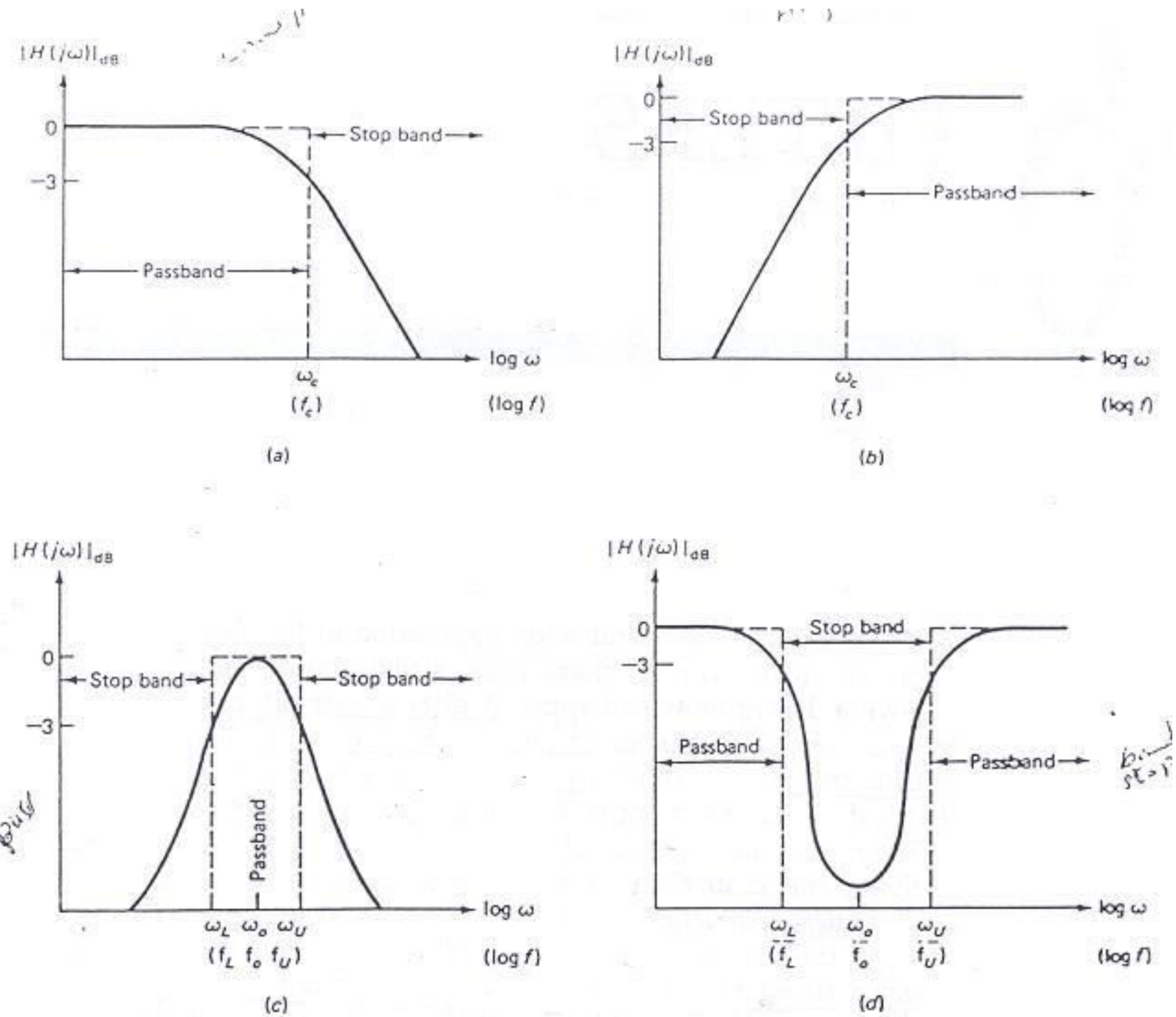
Active Filters

- Op amps have wide applications in design of active filters
- Filter is a circuit designed to pass frequencies within a specific range, while rejecting all frequencies that fall outside this range
- Another class of filters are designed to produce an output that is delayed in time or shifted in phase with respect to filter's input
- Passive filters: constructed using only passive components (resistors, capacitors, inductors)
- Active filters: characteristics are augmented using one or more amplifiers; constructed using op amps, resistors, and capacitors only
  - Allow many filter parameters to be adjusted continuously and at will

Filter Fundamentals

- Five basic types of filters
  - Low-pass (LP)
  - High-pass (HP)
  - Bandpass (BP)
  - Bandstop (notch or band-reject)
  - All-pass (or time-delay)

Response Curves



- ω is in rad/s
- l H(jω) l denotes frequency-dependent voltage gain of filter
- Complex filter response is given by

H(jω) = l H(jω) l <θ(jω)

- If signal frequencies are expressed in Hz, filter response is expressed as l H(jf) l

Filter Terminology

- Filter passband: range of frequencies a filter will allow to pass, either amplified or relatively unattenuated
- All other frequencies are considered to fall into filter's stop band(s)
- Frequency at which gain of filter drops by 3.01 dB from that of passband determines where stop band begins; this frequency is called corner frequency (f_c)
- Response of filter is down by 3 dB at corner frequency (3 dB decrease in voltage gain translates to a reduction of 50% in power delivered to load driven by filter)
- f_c is often called half-power point

Filter Terminology

- Decibel voltage gain is actually intended to be logarithmic representation of power gain
- Power gain is related to decibel voltage gain as
  - $A_P = 10 \log (P_0/P_{in})$
  - $P_0 = (V_0^2/Z_L)$ and $P_{in} = (V_{in}^2/Z_{in})$
  - $A_P = 10 \log [(V_0^2/Z_L) /(V_{in}^2/Z_{in})]$
  - $A_P = 10 \log (V_0^2 Z_{in} /V_{in}^2 Z_L)]$
  - If $Z_L = Z_{in}$, $A_P = 10 \log (V_0^2/V_{in}^2) = 10 \log (V_0/V_{in})^2$
  - $A_P = 20 \log (V_0/V_{in}) = 20 \log A_v$
- When input impedance of filter equals impedance of load being driven by filter, power gain is dependent on voltage gain of circuit only

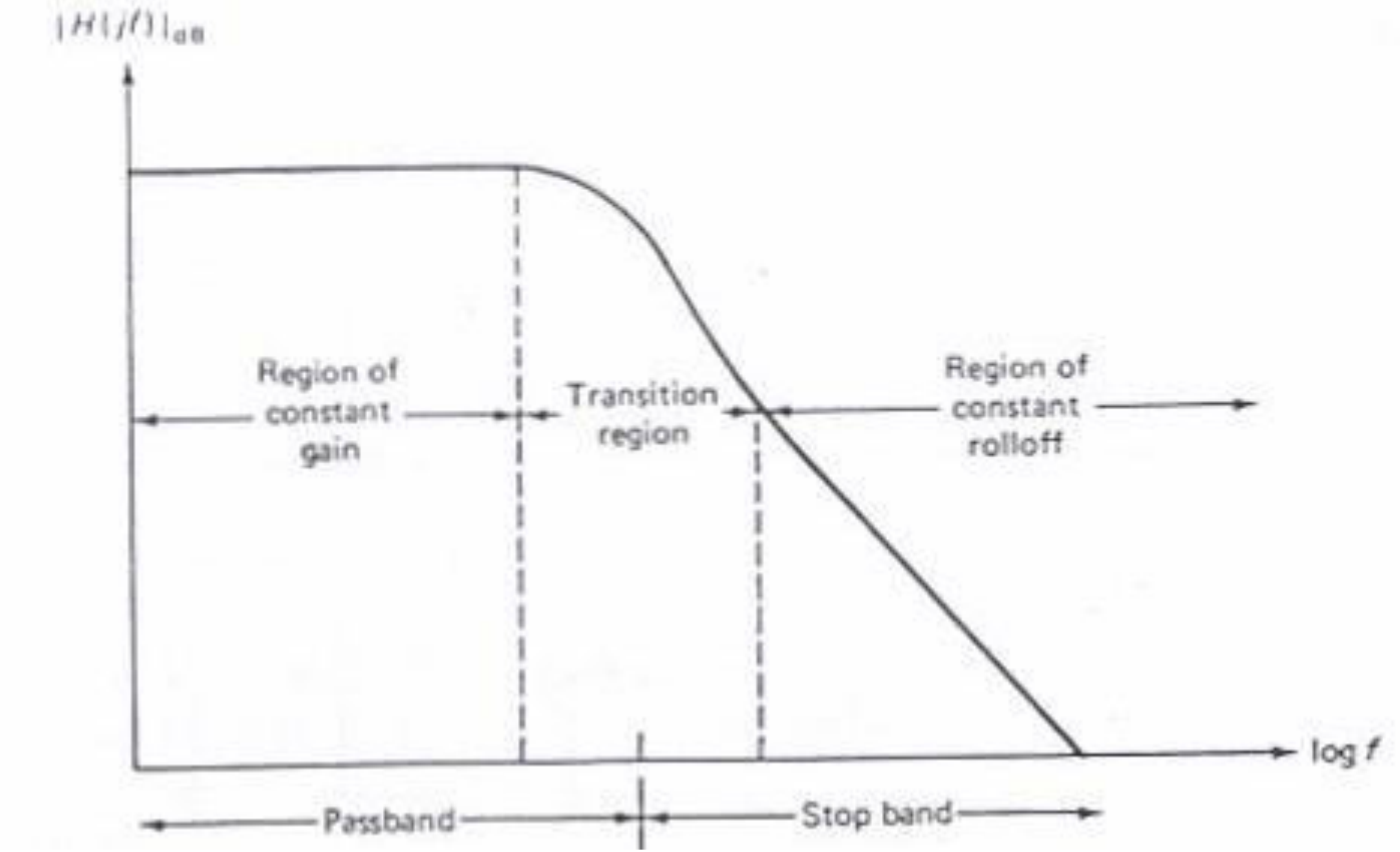
Filter Terminology

- Since we are working with voltage ratios, gain is expressed as voltage gain in dB
  - $| H(j\omega) |_{dB} = 20 \log (V_0/V_{in}) = 20 \log A_V$
- Once frequency is well into stop band, rate of increase of attenuation is constant (dB/decade rolloff)
- Ultimate rolloff rate of a filter is determined by order of that filter
- 1^{st} order filter: rolloff of -20 dB/decade
- 2^{nd} order filter: rolloff of -40 dB/decade
- General formula for rolloff = -20n dB/decade (n is the order of filter)
- Octave is a twofold increase or decrease in frequency
- Rolloff = -6n dB/octave (n is order of filter)

Filter Terminology

- Transition region: region between relatively flat portion of passband and region of constant rolloff in stop band
- Give two filter of same order, if one has a greater initial increase in attenuation in transition region, that filter will have a greater attenuation at any given frequency in stop band
- Damping coefficient (α): parameter that has great effect on shape of LP or HP filter response in passband, stop band, and transition region (0 to 2)
- Filters with lower α tend to exhibit peaking in passband (and stopband) and more rapid and radically varying transition-region response attenuation
- Filters with higher α tend to pass through transition region more smoothly and do not exhibit peaking in passband and stopband

LP Filter Response



Filter Terminology

- HP and LP filters have single corner frequency
- BP and bandstop filters have two corner frequencies (f_L and f_U) and a third frequency labeled as f_0 (center frequency)
- Center frequency is geometric mean of f_L and f_U
- Due to log f scale, f_0 appears centered between f_L and f_U

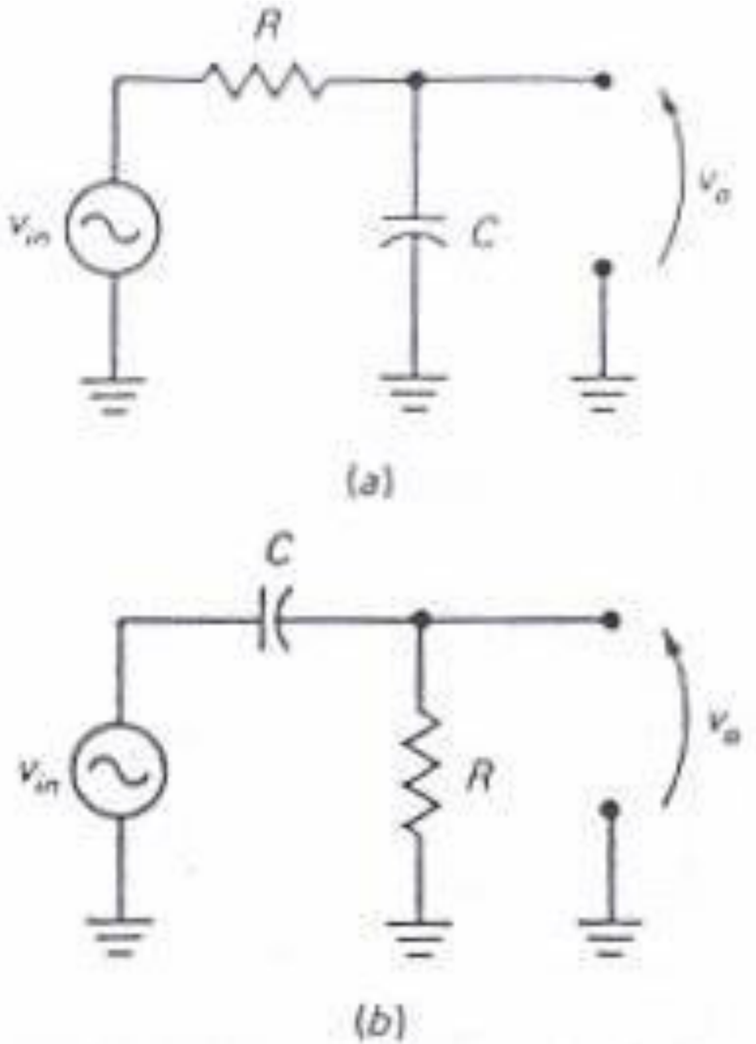
$$f_0 = \text{sqrt}\,(f_L f_U)$$

- Bandwidth of BP or bandstop filter is

$$BW = f_U – f_L$$

- Also, $Q = f_0 / BW$ (BP or bandstop filters)
- BP filter with high Q will pass a relatively narrow range of frequencies, while a BP filter with lower Q will pass a wider range of frequencies
- BP filters will exhibit constant ultimate rolloff rate determined by order of the filter

Basic Filter Theory Review



- Simplest filters are 1st order LP and HP RC sections
  - Passband gain slightly less than unity
- Assuming neglegible loading, amplitude response (voltage gain) of LP section is

  $H(j\omega) = (jX_C) / (R + jX_C)$

  $H(j\omega) = X_C/\text{sqrt}(R^2+X_C^2) <\!-\tan^{-1} (R/X_C)$
- Corner frequency f_c for 1st order LP or HP RC section is found by making $R = X_C$ and solving for frequency

  $R = X_C = 1/(2\pi fC)$

  $1/f_C = 2\pi RC$

  $f_C = 1/(2\pi RC)$
- Gain (in dB) and phase response of 1st order LP

  $H(jf)\text{ dB} = 20 \log [1/\{\text{sqrt}(1+(f/f_c)^2\}] <\!-\tan^{-1} (f/f_C)$
- Gain (in dB) and phase response of 1st order HP

  $H(jf)\text{ dB} = 20 \log [1/\{\text{sqrt}(1+(f_c/f)^2\}] <\tan^{-1} (f_C/f)$